

Large Temperature and Pressure Differences*

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They have reported measurements of liquid He II flowing through narrow slits with pressure differences as large as 1°K. For these cases the linear two-fluid equations are not verified; integration over the temperature paper extends the analysis of the linear equations are no longer valid. The Gorter-Mellink nonlinear theory based on the concept of mutual friction is used; integrals for heat flow and fountain pressure are evaluated using a high-speed computer and the results compared with the linear solutions. The comparison is quite good. It is shown that other values of $A(T)$ are not supported by the explanation in terms of the vortex model near T_λ . Despite the agreement obtained here, several as yet unexplained phenomena in small slits; certainly, most notably the criteria for the

DISCUSSION

the linear equations of motion for pressure and pressure differences in slits, have been reported in two papers (I and II). In interpreting these measurements, linear equations of motion over the slits were used. This approach proved inadequate for pressure and heat flow over a range of temperatures that could not be accounted for by the linear theory. The United States Atomic Energy Commission, Los Alamos, L. I., New York.

linearized nonintegrated theory. However, at sufficiently high heat flows saturation effects appeared producing significant deviations from the predictions of the linear theory.

In this paper we shall discuss the relationship of measurements involving very large heat current densities to solutions of the Gorter-Mellink (3) nonlinear thermohydrodynamical equations. The integrated nonlinear equations are found to reduce to the linear equations for small heat flows. For larger heat currents the calculations using Vinen's (4) values of $A(T)$ in the mutual friction term are in good quantitative agreement with the observations, except in the neighborhood of the λ -point. However, since the Vinen model of dissipation in He II resulting from vortex line turbulence in the superfluid as applied to the present experimental arrangement predicts the breakdown of the equations near the λ -point, the observed deviations may be considered as qualitative support for the theory.

II. DERIVATION OF INTEGRATED FLOW EQUATIONS

A. DERIVATION OF ∇P AND ∇T

In order to obtain solutions to the thermohydrodynamic equations of motion for He II which are applicable to long narrow slits and capillaries, we begin with the following two-fluid equations of motion, including mutual friction¹:

$$\rho_s \frac{D\mathbf{v}_s}{Dt} = -\left(\frac{\rho_s}{\rho}\right) \nabla P + \rho_s s \nabla T - \mathbf{F}_{sn} \quad (1)$$

$$\rho_n \frac{D\mathbf{v}_n}{Dt} = -\left(\frac{\rho_n}{\rho}\right) \nabla P - \rho_n s \nabla T + \mathbf{F}_{sn} - \eta_n \nabla \times \nabla \times \mathbf{v}_n + (2\eta_n + \eta') \nabla (\nabla \cdot \mathbf{v}_n) \quad (2)$$

where

$$\frac{D\mathbf{v}}{Dt} = \frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v} \cdot \nabla) \mathbf{v}.$$

Here the subscripts s and n refer to the superfluid and the normal fluid, η_n is the normal fluid viscosity, and η' is the bulk or second viscosity. The frictional force $\mathbf{F}_{sn}$ accounts for interaction between the superfluid and the normal fluid. The form of this term will be discussed later. Possible other forces acting separately on the normal fluid and on the superfluid are neglected in this treatment.

¹ The equations of motion have been written in various forms, and the correct form for large velocities and including irreversible processes is still controversial. Equations (1) and (2) originate from the ideas of Tisza (5), Landau (6), London (7), and Gorter and Mellink (3), and are believed to serve the present purposes well to a good first approximation. The more detailed treatment of the second viscosity terms by Khalatnikov (8) is necessary for analyzing experiments on such phenomena as first and second sound; but in experiments on fountain pressure and heat conduction the second viscosity plays a subordinate role and the following more easily handled equations suffice.